Use of the defining equations of the evolute gives

$$F_R = nR^{n-1} = n(S_1 + R_0)^{n-1},$$

$$F_S = -\ mCS^{m-1} = -\ mC^{1/m}(S_1 + R_0)^{n(m-1)/m}.$$

From these values of F_S and F_R the equation of the evolute is found, from (2), to be

$$nR_1(S_1 + R_0)^{n-1} - mC^{1/m}(S_1 + R_0)(S_1 + R_0)^{n(m-1)/m} = 0.$$

An easy simplification of this equation reduces it to

$$R_1{}^m = C\left(\frac{m}{n}\right)^m (S_1 + R_0)^{2m-n}. \tag{6}$$

If this last equation is to represent a curve similar to the base curve (5), then it is again necessary—first, that $R_0 = 0$, *i.e.*, that a cusp exist on the base curve; and second, that the exponents correspond. This gives $m = n$. Hence[1] the result is obtained;

All curves whose intrinsic equations can be written $R^n = CS^m$ have evolutes similar to themselves only when $R_0 = 0$ and when $m = n$. This gives the logarithmic spiral $R^m = CS^m$.

II. Functions of Half-Angles of a Triangle.

By R. D. Bohannan, Ohio State University.

In the accompanying figure I is the center of the circle inscribed in the triangle ABC, and E is the center of the escribed circle touching the side BC and the prolongations of AB and AC. IH, IK, EF, EG are radii of these circles drawn to some of the points of tangency. If $BC = a$, $AC = b$, $AB = c$, $a + b + c = 2s$, then

$$AH = s - a; \qquad AG = s; \qquad CK = s - c; \qquad BK = s - b.$$

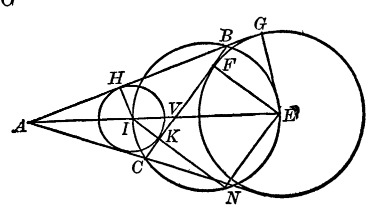
A circle on IE as diameter passes through C, B. If IK is prolonged to meet this circle in N, KN is equal to FE.

$$\tan\frac{A}{2} = \frac{IH}{AH} = \frac{EG}{AG}.$$

Hence

$$\tan^2\frac{A}{2} = \frac{IH}{AH}\cdot\frac{EG}{AG} = \frac{IH\cdot EG}{s(s-a)}$$

$$= \frac{IK\cdot FE}{s(s-a)} = \frac{IK\cdot KN}{s(s-a)}$$



$$= \frac{CK\cdot KB}{s\cdot(s-a)} = \frac{(s-b)(s-c)}{s(s-a)}.$$

[1] The cases excluded by the assumptions $m \neq 0$, $n \neq 0$, $C \neq 0$ in the course of the proof can be shown as before to yield nothing new.—Editor.

Let V be the intersection of AE and BC. Then

$$\sin^2\frac{A}{2}=\frac{IH}{AI}\cdot\frac{EG}{AE},\qquad \cos^2\frac{A}{2}=\frac{AH}{AI}\cdot\frac{AG}{AE}.$$

Since BI and CE (not drawn in the figure) are angle-bisectors,

$$\frac{AI}{IV}=\frac{AB}{BV},\qquad \frac{AE}{VE}=\frac{AC}{VC}.$$

But

$$IV\cdot VE=BV\cdot VC.$$

Hence

$$AI\cdot AE=AB\cdot AC=bc.$$

Therefore

$$\sin^2\frac{A}{2}=\frac{(s-b)(s-c)}{bc},\qquad \cos^2\frac{A}{2}=\frac{s(s-a)}{bc}.$$

III. The Use of the Vector in Analytical Geometry.

By Vincent C. Poor, University of Michigan.

In a first course in analytical geometry it is necessary to adhere rather closely to the text, whatever the text. This is true not because of the nature of the subject matter but because of the mathematical immaturity of the students electing the subject. Important innovations thus furnish one excuse for another textbook.

In many of the textbooks on analytical geometry the directed line is not mentioned at all. This is deplorable from the point of view of the physicist, for the geometric interpretation of many physical quantities leads to simplicity in expression and clearness in comprehension. Aside from this need the subject of analytical geometry may, in my opinion, be much more easily and directly presented if a more extended use of the vector be made.

The ground work for this is to be found in some of our textbooks, *e.g.*, Woods and Bailey, *A Course in Mathematics*, Vol. I; Ziwet and Hopkins, *Analytic Geometry*. In their study of directed lines we find the equivalents of the following theorems:

Theorem I. *The projection of a line segment on another line is equal to the length of the line segment into the cosine of their included angle.*

Theorem II. *The projection of a broken line on another line is equal to the projection of the join of its end points.*

In a number of the books the fundamental theorem of the geometry of segments is deduced, namely

Theorem III. *Given three points, O, P, Q, on a directed line, then*

$$PQ=OQ-OP,$$

in magnitude and sense.

If O is taken as the origin and the coördinates x_1 and x_2 be assigned to the